Please check the examination details below before entering your candidate information

| Candidate surname | Other names |
|---|---|
| | |

Centre Number

Candidate Number

**Pearson Edexcel International Advanced Level**

# Tuesday 14 May 2024

Morning (Time: 1 hour 30 minutes)

**Paper reference** **WMA12/01**

# Mathematics

## International Advanced Subsidiary/Advanced Level
## Pure Mathematics P2

**You must have:**
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

*Turn over*

**1.** (a) Find the first four terms, in ascending powers of $x$, of the binomial expansion of

$$\left(1 - \frac{1}{6}x\right)^9$$

giving each term in simplest form.

**(3)**

(b) Hence find the coefficient of $x^3$ in the expansion of

$$(10x + 3)\left(1 - \frac{1}{6}x\right)^9$$

giving the answer in simplest form.

**(2)**

a. Use the binomial expansion formula to expand $\left(1-\frac{1}{6}x\right)^9$

BINOMIAL EXPANSION FORMULA:
$$(a+bx)^n = a^n + \binom{n}{1}a^{n-1}(bx)^1 + \binom{n}{2}a^{n-2}(bx)^2 + \ldots + \binom{n}{n-1}a^1(bx)^{n-1} + (bx)^n$$

$$\left(1-\frac{1}{6}x\right)^9 = 1^9 + \binom{9}{1}(1)^{9-1}\left(-\frac{1}{6}x\right)^1 + \binom{9}{2}(1)^{9-2}\left(-\frac{1}{6}x\right)^2 + \binom{9}{3}(1)^{9-3}\left(-\frac{1}{6}x\right)^3$$ Only requires the first 4 terms

$$= 1 + \binom{9}{1}(1)^8\left(-\frac{1}{6}x\right) + \binom{9}{2}(1)^7\left(-\frac{1}{6}\right)^2 + \binom{9}{3}(1)^6\left(-\frac{1}{6}\right)^3$$

$$= 1 - \frac{3}{2}x + x^2 - \frac{7}{18}x^3$$

$$\boxed{\left(1-\frac{1}{6}x\right)^9 = 1 - \frac{3}{2}x + x^2 - \frac{7}{18}x^3}$$

b. Since we want the $x^3$ term of $(10x+3)\left(1-\frac{1}{6}x\right)^9$ we need to find all the COMBINATIONS of terms which multiply to $x^3$

| | $1$ | $-\frac{3}{2}x$ | $x^2$ | $-\frac{7}{18}x^3$ |
|---|---|---|---|---|
| $10x$ | $10x$ | $-15x$ | $10x^3$ | $-\frac{35}{9}x^4$ |
| $3$ | $3$ | $-\frac{9}{2}x$ | $3x^2$ | $-\frac{7}{6}x^3$ |

For these types of questions, we can just use our expansion from part a, thus the word 'hence' in the question. We can see only $10x \cdot x^2$ and $3 \cdot -\frac{7}{18}x^3$ create an $x^3$ term. However, still check if there is anymore terms that multiply to $x^3$.

$x^3$ term: $10x^3 - \frac{7}{6}x^3 = \frac{53}{6}x^3$

$$\boxed{\therefore \text{coefficient of } x^3 \text{ term}\cdot \frac{56}{6}}$$

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**Question 1 continued**

**(Total for Question 1 is 5 marks)**

**2.** **In this question you must show all stages of your working.**
**Solutions relying entirely on calculator technology are not acceptable.**

In an arithmetic series,

- the sixth term is 2
- the sum of the first ten terms is –80

For this series,

(a) find the value of the first term and the value of the common difference.
**(4)**

(b) Hence find the smallest value of $n$ for which

$$S_n > 8000$$
**(3)**

a. Use the formulae of the arithmetic sequence and the sum of its $n^{th}$ terms to form a simultaneous equation in terms of a and d.

ARITHMETIC SEQUENCE FORMULA

$a_n = a + (n-1)d$

SUM OF NTH TERMS OF ARITHMETIC SEQUENCE

$S_n = \frac{n}{2}(2a + (n-1)d)$

· Sixth term is 2

$2 = a + (6-1)d$

① $a + 5d = 2$

· Sum of first 10 terms = -80

$-80 = \frac{10}{2}(2a + (10-1)d)$

$= 5(2a + 9d)$

② $10a + 45d = -80$

$10 \cdot ① - ②$ · ← We multiply equation ① by 10 in order have equal coefficients of a with ②

$10a + 50d = 20$

$-\ 10a + 45d = -80$

$5d = 100$

$d = 20$

Substitute $d=20$ into ① to find a

$a + 5(20) = 2$

$a + 100 = 2$

$a = -98$

$\therefore a = -98,\ d = 20$

b. Use the values of a and d from part a.

$S_n > 8000$

$\frac{n}{2}(2(-98) + (n-1)(20)) > 8000$

$\frac{n}{2}(-196 + 20n - 20) > 8000$

$\frac{n}{2}(20n - 216) > 8000$

$10n^2 - 108n - 8000 > 0$

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**Question 2 continued**

$5n^2 - 54n - 4000 > 0$

Solve for n using quadratic formula.

QUADRATIC FORMULA

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$n > \frac{-(-54) \pm \sqrt{(-54)^2 - 4(5)(-4000)}}{2(5)}$$

$n > 34.195$ , $\cancel{n > -23.395}$ reject negative value of n as this is impossible

↑ round up as n has to be an integer

$\boxed{n = 35}$

**(Total for Question 2 is 7 marks)**

**3.** **In this question you must show all stages of your working.**
**Solutions relying entirely on calculator technology are not acceptable.**

(i) Using the laws of logarithms, solve

$$2\log_2(2-x) = 4 + \log_2(x+10)$$

**(5)**

(ii) Find the value of

$$\log_{\sqrt{a}} a^6$$

where $a$ is a positive constant greater than 1

**(1)**

i. Rewrite equation so that it is only in terms of $\log_2$

$\underbrace{2\log_2(2-x)}_{a\log(b)=\log(b^a)} = 4 + \log_2(x+10)$

$2^4=16$, so $\log_2(16)=4$

$\log_2[(2-x)^2] = \underbrace{\log_2(16) + \log_2(x+10)}_{\log(a)+\log(b)=\log(ab)}$

$\log_2[(2-x)^2] = \log_2[16(x+10)]$ Solve for n using quadratic formula.

Raise both sides to the power of 2 to remove $\log_2$, as $a^{\log_a(x)} = x$

$2^{\log_2[(2-x)^2]} = 2^{\log_2[16(x+10)]}$

$(2-x)^2 = 16(x+10)$

$4-4x+x^2 = 16x+160$

$x^2-20x-156=0$

Solve for $x$ using quadratic formula.

QUADRATIC FORMULA

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \dfrac{-(-20) \pm \sqrt{(-20)^2 - 4(1)(-156)}}{2(1)}$

$x=26$ , $x=-6$

Remember that you cannot log a negative number! (e.g. $\log(-2)$ or $\ln(-6)$ are not possible)

Testing $x=26$

$\log_2(2-x) = \log_2(2-(26))$

$= \log_2(-24)$ ✗

cannot be $x=26$ as it leads to a negative number

Testing $x=-6$

$\log_2(2-x) = \log_2(2-(-6))$

$= \log_2(8)$ ✓

$\log_2(x+10) = \log_2((-6)+10)$

$= \log_2(4)$ ✓

**Question 3 continued**

Since $x=-6$ does not lead to any negative values in log functions:

$\boxed{x=-6}$

ii. $\underbrace{\log_{\sqrt{a}}(a^6)}_{a\log(b)=\log(b^a)} = 6\log_{\sqrt{a}}(a)$ You can rewrite a as $a^{\frac{1}{2}} \times a^{\frac{1}{2}}$

$= \underbrace{6\log_{\sqrt{a}}(a^{\frac{1}{2}} \times a^{\frac{1}{2}})}_{\log(a)+\log(b)=\log(ab)}$

$= 6\left[\log_{\sqrt{a}}(a^{\frac{1}{2}}) + \log_{\sqrt{a}}(a^{\frac{1}{2}})\right]$

$= 6(1+1)$

$= 6(2)$

$\boxed{\log_{\sqrt{a}}(a^6) = 12}$

**(Total for Question 3 is 6 marks)**

**4.** $$f(x) = (x - 2)(2x^2 + 5x + k) + 21$$

where $k$ is a constant.

(a) State the remainder when $f(x)$ is divided by $(x - 2)$ **(1)**

Given that $(2x - 1)$ is a factor of $f(x)$

(b) show that $k = 11$ **(2)**

(c) Hence

(i) fully factorise $f(x)$,

(ii) find the number of real solutions of the equation

$$f(x) = 0$$

giving a reason for your answer.

**(5)**

a. In $f(x)$, we can see that $(x-2)(2x^2+5x+k)$ is a multiple of $(x-2)$ so it will have no remainder when divided

therefore the remainder is: $\boxed{21}$

b. If $(2x-1)$ is a factor to $f(x)$, then according to the FACTOR THEOREM, $f(\frac{1}{2}) = 0$ $(2x-1=0 \rightarrow x=\frac{1}{2})$

$f(\frac{1}{2}) = (\frac{1}{2}-2)(2(\frac{1}{2})^2 + 5(\frac{1}{2}) + k) + 21$

$0 = (\frac{1}{2}-2)(2(\frac{1}{2})^2 + 5(\frac{1}{2}) + k) + 21$

$0 = -\frac{3}{2}(\frac{1}{2} + \frac{5}{2} + k) + 21$

$-21 = -\frac{3}{2}(3 + k)$

$k + 3 = 21$

$k = 11$

$\boxed{k = 11}$

ci. Expand $f(x)$ in order to do algebraic division

$f(x) = (x-2)(2x^2 + 5x + 11) + 21$

$f(x) = 2x^3 + 5x^2 + 11x - 4x^2 - 10x - 22 + 21$

$f(x) = 2x^3 + x^2 + x - 1$

$$\begin{array}{r|l} & x^2 + x + 1 \\ \hline 2x-1 & 2x^3 + x^2 + x - 1 \\ & -(2x^3 - x^2) \\ \hline & \quad 2x^2 + x \\ & \quad -(2x^2 - x) \\ \hline & \qquad 2x - 1 \\ & \qquad -(2x - 1) \\ \hline & \qquad 0 \quad 0 \end{array}$$

**Question 4 continued**

$\therefore f(x) = (2x-1)(x^2+x+1)$

Find the discriminant of $x^2+x+1$

DISCRIMINANT FORMULA

$b^2-4ac = 0 \rightarrow$ Repeated roots

$b^2-4ac > 0 \rightarrow$ Two real roots

$b^2-4ac < 0 \rightarrow$ No real roots

$b^2-4ac = (1)^2-4(1)(1)$

$= 1-4$

$b^2-4ac = -3$

$\therefore$ Since $x^2+x+1$ has discriminant $< 0$, $f(x)=0$ must only have one real solution at $x=\frac{1}{2}$

**Question 4 continued**

**Question 4 continued**

**(Total for Question 4 is 8 marks)**

**5.** **In this question you must show detailed reasoning.**

(a) Given that $x$ and $y$ are positive numbers such that

$$(x-y)^3 > x^3 - y^3$$

prove that

$$y > x$$

**(4)**

(b) Using a counter example, show that the result in part (a) is not true for all real numbers.

**(2)**

a. Expand $(x-y)^3$ and manipulate the equation to derive what needs to be proven

$(x-y)^3 = (x-y)(x-y)(x-y)$

$= (x-y)(x^2-xy-xy+y^2)$

$= (x-y)(x^2-2xy+y^2)$

$= x^3-2x^2y+xy^2-x^2y+2xy^2-y^3$

$(x-y)^3 = x^3-3x^2y+3xy^2-y^3$

$x^3-3x^2y+3xy^2-y^3 > x^3-y^3$

$-3x^2y+3xy^2 > 0$

$3xy(y-x) > 0$

Since $3xy(y-x)>0$ and $x$ and $y$ are positive numbers, $y-x>0 \rightarrow y-x>0$

b COUNTER EXAMPLE: We find values in the set we are given which disproves the statement

Pick suitable real values for $x$ and $y$ as a counter example

$x=3$ $y=-5$ (This is suitable as $y \not> x$ but we are trying to show $(x-y)^3 > x^3-y^3$ still holds despite that)

$(x-y)^3 > x^3-y^3$

LHS: $(x-y)^3 = (3--5)^3 = (8)^3 = 512$

RHS: $x^3-y^3 = 3^3-(-5)^3 = 27--125 = 152$

$\therefore$ $512>152$, but $y>x \rightarrow -5 \not> 3$

**Question 5 continued**

**(Total for Question 5 is 6 marks)**

**6.** (a) Sketch the curve with equation

$$y = a^x + 4$$

where $a$ is a positive constant greater than 1

On your sketch, show

- the coordinates of the point of intersection of the curve with the $y$-axis
- the equation of the asymptote of the curve

**(3)**

| $x$ | 2 | 2.3 | 2.6 | 2.9 | 3.2 | 3.5 |
|---|---|---|---|---|---|---|
| $y$ | 0 | 0.3246 | 0.8629 | 1.6643 | 2.7896 | 4.3137 |

The table shows corresponding values of $x$ and $y$ for

$$y = 2^x - 2x$$

with the values of $y$ given to 4 decimal places as appropriate.

Using the trapezium rule with all the values of $y$ in the given table,

(b) obtain an estimate for $\int_2^{3.5} \left(2^x - 2x\right) \mathrm{d}x$, giving your answer to 2 decimal places.

**(3)**

(c) Using your answer to part (b) and making your method clear, estimate

(i) $\int_2^{3.5} \left(2^x + 2x\right) \mathrm{d}x$

(ii) $\int_2^{3.5} \left(2^{x+1} - 4x\right) \mathrm{d}x$

**(3)**

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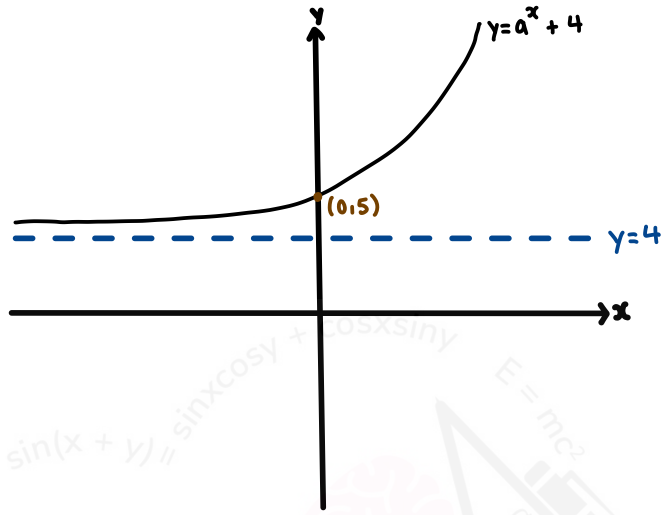
**Question 6 continued**

a $y = a^x + 4$ is an exponential function, so it has the same shape as a $y = e^x$ graph which has an asymptote at $y=0$.

The graph is also being translated by $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$, indicated by the +4 in the equation




b. TRAPEZIUM RULE

$$\int_a^b y\,dx = \frac{1}{2}h\left[(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\right], \text{ where } h = \frac{b-a}{n}$$

represents the width of each strip

height of the first and last strip respectively

heights of strips in between the first and last

number of strips

$h = \frac{3.5-2}{5}$

$h = 0.3$ The table gives us six y-values, indicating that 5 strips are being used.

$$\therefore \int_2^{3.5} 2^x - 2x\,dx = \frac{1}{2}(0.3)\left[(0 + 4.3137) + 2(0.3246 + 0.8629 + 1.6643 + 2.7896)\right]$$

$= 2.339475$

$$\boxed{\int_2^{3.5} 2^x - 2x\,dx = 2.34 \text{ (2dp)}}$$

ci. We can rewrite the integral in order to get part of it in terms of the result in part b.

$$\int_2^{3.5} 2^x + 2x\,dx = \int_2^{3.5} 2^x - 2x + 4x\,dx$$

The integral in part b → $= \int_2^{3.5} 2^x - 2x\,dx + \int_2^{3.5} 4x\,dx$ ← This is very simple to integrate

$= 2.339475 + \left[2x^2\right]_2^{3.5}$

$= 2.339475 + 16.5 = \boxed{18.84 \text{ (2dp)}}$



P 7 5 7 0 8 A 0 1 5 3 2

**Question 6 continued**

cii. Very similar idea to part ci, but we also use the index law $a^x \times a^y = a^{x+y}$

$$\int_2^{3.5} 2^{x+1} - 4x \, dx = \int_2^{3.5} \underbrace{2(2^x}_{2\times 2^x = 2^{x+1}} - 2x) \, dx$$ The integral in part b

$$= 2\int_2^{3.5} 2^x - 2x \, dx$$

$$= 2 \times 2.339475$$

$$\boxed{= 4.68 \text{ (2dp)}}$$

**Question 6 continued**

**(Total for Question 6 is 9 marks)**

7. The circle $C_1$ has equation

$$x^2 + y^2 + 8x - 10y = 29$$

(a) (i) Find the coordinates of the centre of $C_1$

(ii) Find the exact value of the radius of $C_1$

**(3)**

**In part (b) you must show detailed reasoning.**

The circle $C_2$ has equation

$$(x-5)^2 + (y+8)^2 = 52$$

(b) Prove that the circles $C_1$ and $C_2$ neither touch nor intersect.

**(3)**

ai. We can complete the square to get the equation of $C_1$ in the form $(x-a)^2 + (y-a)^2 = r^2$, r-radius, (a, b)-centre

COMPLETING THE SQUARE FORMULA
$ax^2 + bx + c = a\left[(x+\frac{b}{2a})^2 - (\frac{b}{2a})^2\right] + c$

$x^2 + y^2 + 8x - 10y = 29$

$x^2 + 8x + y^2 - 10y = 29$ We rewrite the equation to group together the x and y.

$(x+4)^2 - 16 + (y-5)^2 - 25 = 29$ Apply the formula above to complete the square

$(x+4)^2 + (y-5)^2 = 70$

$\therefore$ The centre is $(-4, 5)$

aii. Using the equation of $C_1$ obtained in part ai:

radius: $r = \sqrt{70}$

b For two circles to intersect, the distance between their centres must be less than or equal to the sum of their radii.

$C_1$ centre: $(-4, 5)$ radius: $\sqrt{70}$ (from part a)

$C_2$ centre: $(5, -8)$ radius: $\sqrt{52}$

DISTANCE BETWEEN TWO POINTS FORMULA
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Distance between centres of $C_1$ and $C_2 = \sqrt{(-4-5)^2 + (5--8)^2}$

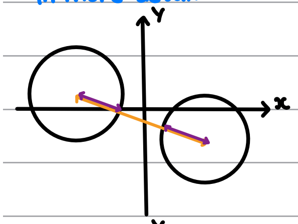
$= 5\sqrt{10} \simeq 15.811$ (3dp)

Sum of radii $= \sqrt{70} + \sqrt{52} \simeq 15.578$ (3dp)

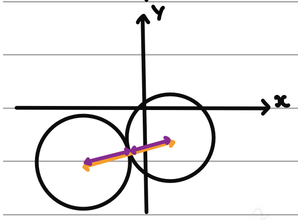
**Question 7 continued**

$\therefore$ Since the sum of the radii is less than the distance between the centres $15.578 < 15.811$, $C_1$ and $C_2$ do not touch or intersect.

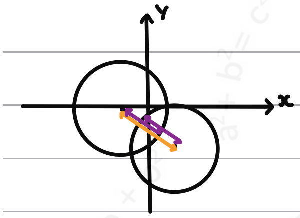
In more detail:




Visually, the sum of the radii of the circles is less than the distance between their centres, so they do not intersect.




Sum of the radii = Distance between their centres, so circles intersect once




Sum of the radii > Distance between their centres, so circles intersect twice

**Question 7 continued**

**Question 7 continued**

**(Total for Question 7 is 6 marks)**

**8.** **In this question you must show all stages of your working.**
**Solutions relying entirely on calculator technology are not acceptable.**

(i) Solve, for $0 < x \leqslant \pi$, the equation

$$5\sin x \tan x + 13 = \cos x$$

giving your answer in radians to 3 significant figures.

**(5)**

(ii) The temperature inside a greenhouse is monitored on one particular day.

The temperature, $H$°C, inside the greenhouse, $t$ hours after midnight, is modelled by the equation

$$H = 10 + 12\sin(kt + 18)° \qquad 0 \leqslant t < 24$$

where $k$ is a constant.

**Use the equation of the model to answer parts (a) to (c).**

Given that

- the temperature inside the greenhouse was 20 °C at 6 am
- $0 < k < 20$

(a) find all possible values for $k$, giving each answer to 2 decimal places.

**(4)**

Given further that $0 < k < 10$

(b) find the maximum temperature inside the greenhouse,

**(1)**

(c) find the time of day at which this maximum temperature occurs.

Give your answer to the nearest minute.

**(2)**

i. $5\sin x \underbrace{\tan x}_{} + 13 = \cos x$

$\tan x = \dfrac{\sin x}{\cos x}$

$5\sin x \cdot \dfrac{\sin x}{\cos x} + 13 = \cos x$

$5\underbrace{\sin^2 x}_{} + 13\cos x = \cos^2 x$ Multiply both sides by $\cos x$ to remove the $\cos x$ denominator

$\sin^2 x = 1 - \cos^2 x$

$5(1-\cos^2 x) + 13\cos x = \cos^2 x$ Notice we can obtain a quadratic equation in terms of $\sin x$ or $\cos x$. Let's choose one in terms of $\cos x$, so

$5 - 5\cos^2 x + 13\cos x = \cos^2 x$ substitute $1-\cos^2 x$ into $\sin^2 x$

$6\cos^2 x - 13\cos x - 5 = 0$ Solve for $\cos x$ using quadratic formula.

QUADRATIC FORMULA

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

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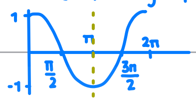
**Question 8 continued**

$\cos x = \dfrac{-(-13) \pm \sqrt{(-13) - 4(6)(-5)}}{2(6)}$

$\cos x = -\frac{1}{3}$ , $\cancel{\cos x = \frac{5}{2}}$ Reject $\cos x = \frac{5}{2}$ as the range of $\cos x$ is $-1 \leq \cos x \leq 1$

$\cos x = -\frac{1}{3}$

$\boxed{x = 1.91 \text{ (2dp)}}$ Notice the range of $x$: $0 \leq x \leq \pi$. Because of this we can be sure there is only one solution for $x$.




iia. Substitute in the information given to solve for k

$H = 20$ when $t = 6$

$H = 10 + 12\sin(kt + 18)°$

$20 = 10 + 12\sin(6k + 18)$

$10 = 12\sin(6k + 18)$

$\sin(6k + 18) = \frac{10}{12}$

$6k + 18 = 56.44.., 123.55...$

$k = 6.407.., 17.59...$

$\boxed{k = 6.41, 17.6 \text{ (1dp)}}$ These are the only solutions for k due to the range $0 < k < 20$

iib. The maximum temperature is when $\sin(kt + 18) = 1$

$H = 10 + 12\sin(kt + 18)$

$H = 10 + 12 \times 1$

$\boxed{H = 22°c}$

iic. Since $0 < k < 10$, $k = 6.407..$

$\sin(kt + 18) = 1$

$\sin(6.407t + 18) = 1$

$6.407t + 18 = 90$

$t = 11.237...$

11 hours, $(0.237.. \times 60)$ minutes

$\boxed{= 11:14}$ We know there is only one time as the question subtly hints at a singular time of day



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**Question 8 continued**

**Question 8 continued**

**(Total for Question 8 is 12 marks)**

**9.**

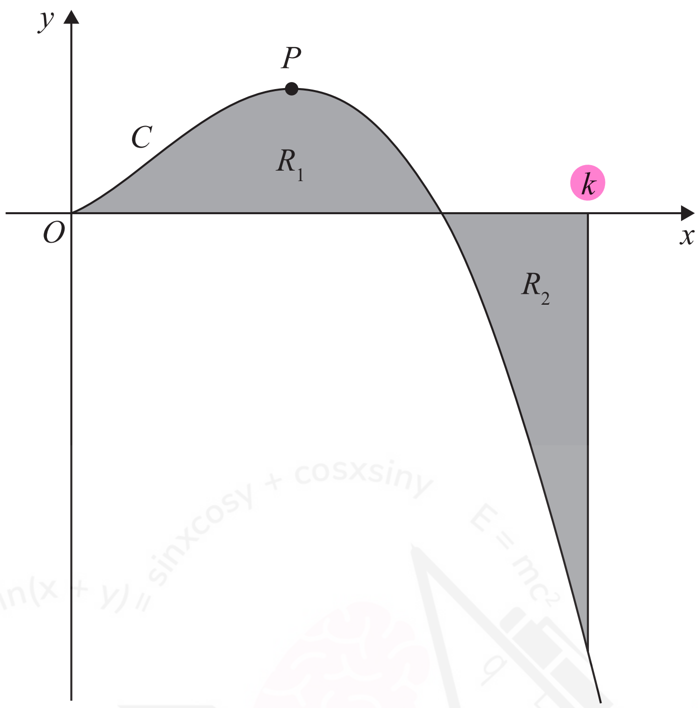


**Figure 1**

Figure 1 is a sketch of the curve $C$ with equation

$$y = 2x^{\frac{3}{2}}(4 - x) \qquad x \geqslant 0$$

The point $P$ is the stationary point of $C$.

(a) Find, using calculus, the $x$ coordinate of $P$.

**(4)**

The region $R_1$, shown shaded in Figure 1, is bounded by $C$ and the $x$-axis.

The region $R_2$, also shown shaded in Figure 1, is bounded by $C$, the $x$-axis and the line with equation $x = k$, where $k$ is a constant.

Given that the area of $R_1$ is equal to the area of $R_2$

(b) find, using calculus, the exact value of $k$.

**(4)**

a. At a stationary point, $\frac{dy}{dx} = 0$

$y = 2x^{\frac{3}{2}}(4-x)$

$y = \underbrace{8x^{\frac{3}{2}} - 2x^{\frac{5}{2}}}$

$\frac{d}{dx}(ax^n) = anx^{n-1}$

$\frac{dy}{dx} = 12x^{\frac{1}{2}} - 5x^{\frac{3}{2}}$

$12x^{\frac{1}{2}} - 5x^{\frac{3}{2}} = 0$

$12x^{\frac{1}{2}} = 5x^{\frac{3}{2}}$





DO NOT WRITE IN THIS AREA

**Question 9 continued**

$\boxed{x = \frac{12}{5}}$

b. Since area of $R_1$ = area of $R_2$, $\int_0^k 2x^{\frac{3}{2}}(4-x)\,dx = 0$ This is because $R_1$ is above the $x$-axis and $R_2$ is below, so $R_2$ is negative and cancels out $R_1$

$$\int_0^k 2x^{\frac{3}{2}}(4-x)\,dx = \underbrace{\int_0^k 8x^{\frac{3}{2}} - 2x^{\frac{5}{2}}\,dx}_{\int_\alpha^\beta ax^n\,dx = \left[\frac{a}{n+1}x^{n+1}\right]_\alpha^\beta}$$

$$0 = \left[\frac{16}{5}x^{\frac{5}{2}} - \frac{4}{7}x^{\frac{7}{2}}\right]_0^k$$

$$0 = \frac{16}{5}k^{\frac{5}{2}} - \frac{4}{7}k^{\frac{7}{2}}$$

$$\frac{16}{5}k^{\frac{5}{2}} = \frac{4}{7}k^{\frac{7}{2}}$$

$$\frac{k^{\frac{7}{2}}}{k^{\frac{5}{2}}} = \frac{16}{7} \div \frac{4}{7}$$

$$\boxed{k = \frac{28}{5}}$$

**Question 9 continued**

**Question 9 continued**

**(Total for Question 9 is 8 marks)**

**10.** **In this question you must show all stages of your working.**
**Solutions relying entirely on calculator technology are not acceptable.**

The number of dormice and the number of voles on an island are being monitored.

Initially there are 2000 dormice on the island.

A model predicts that the number of dormice will increase by 3% each year, so that the numbers of dormice on the island at the end of each year form a geometric sequence.

(a) Find, according to the model, the number of dormice on the island 6 years after monitoring began. Give your answer to 3 significant figures.

**(2)**

The number of voles on the island is being monitored over the same period of time.

Given that

- 4 years after monitoring began there were 3690 voles on the island
- 7 years after monitoring began there were 3470 voles on the island
- the number of voles on the island at the end of each year is modelled as a geometric sequence

(b) find the equation of this model in the form

$$N = ab^t$$

where $N$ is the number of voles, $t$ years after monitoring began and $a$ and $b$ are constants. Give the value of $a$ and the value of $b$ to 2 significant figures.

**(3)**

When $t = T$, the number of dormice on the island is equal to the number of voles on the island.

(c) Find, according to the models, the value of $T$, giving your answer to one decimal place.

**(3)**

a. Use the formula for the $n^{th}$ term of a geometric sequence

NTH TERM OF GEOMETRIC SEQUENCE

$a_n = ar^{n-1}$

first term, common ratio

$a = 2000$ $r = 1.03$ ← 3% increase per year

$a_n = 2000(1.03)^{n-1}$

Remember, the first term in the sequence is the $0^{th}$ year, so six years after monitoring started, the number of dormice is equal to the $7^{th}$ term in the sequence

Six years after monitoring started: $a_7 = 2000(1.03)^{7-1}$

$a_7 = 2388.104..$

$\boxed{a_7 = 2390 \text{ (3sf)}}$

**Question 10 continued**

b. Same idea as in part a, the $4^{th}$ and $7^{th}$ year of monitoring are the $5^{th}$ and $8^{th}$ terms respectively

$ar^{5-1} = 3690 \qquad ar^{8-1} = 3470$

① $ar^4 = 3690$ ② $ar^7 = 3470$

② ÷ ①

$$\frac{ar^7}{ar^4} = \frac{3470}{3690}$$

$$r^3 = \frac{347}{369}$$

$$r = 0.9797\ldots$$

Substitute $r = 0.9797\ldots$ in ① to solve for a.

$$a(0.9797\ldots)^4 = 3690$$

$$a = 4005.1816\ldots$$

In the equation given in the question: $N = ab^t$, a is 4000 and b is 0.98 (2sf)

$$\boxed{\therefore N = 4000(0.98)^t}$$

c. Set the sequences obtained in part a and b equal to each other.

$$2000(1.03)^T = 4000(0.98)^T$$

$$\frac{(1.03)^T}{(0.98)^T} = \frac{4000}{2000}$$

$$\left(\frac{1.03}{0.98}\right)^T = 2$$

$$\underbrace{\log\left[\left(\frac{1.03}{0.98}\right)^T\right]}_{a\log(b) = \log(b^a)} = \log 2$$ Log both sides to bring T down.

$$T\log\left(\frac{1.03}{0.98}\right) = \log 2$$

$$T = 13.929\ldots$$

$$\boxed{T = 13.9 \text{ (1dp)}}$$



P 7 5 7 0 8 A 0 3 1 3 2

**Question 10 continued**

**(Total for Question 10 is 8 marks)**

**TOTAL FOR PAPER IS 75 MARKS**